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Microscale Yielding as Mechanism for Low-frequency Intrinsic Seismic Wave Attenuation

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SUMMARY

We revisit the idea of microscale yielding being responsible for attenuation of small amplitude waves in a wide frequency range. We consider microscopic rate-independent irreversible deformation around cavities causing local stress amplification in pre-stressed porous media as a mechanism responsible for frequency independent attenuation. Following the effective media approach, we consider low porosity material containing non-interacting isolated spherical pores under cyclic loading by isotropic stress field imitating passage of a wave, and evaluate resulting dissipation in terms of quality factor Q. Assuming initial local microscopic stress state around the cavity at the yield, we show that even for small seismic strains attenuation can be high and independent of both frequency and strain amplitude.

Introduction. Energy dissipation accompanying the wave propagation process results in attenuation of waves over a broad frequency range. Significant effort during the past years was focused on quantification of attenuation properties as well as on the attempts to identify the mechanism responsible for the phenomenon. Observations show that for the low-frequency seismic waves the specific attenuation factor 1/Q is nearly independent of frequency regardless of a fluid saturation. In very dry materials 1/Q is independent of frequency over the whole range of frequencies (Knopoff, 1964; Kibblewhite, 1989). Several major possible mechanisms of attenuation were proposed: matrix anelasticity; attenuation due to viscosity and flow of saturating fluids; patchy and partial saturation effects; energy absorbed in systems undergoing phase changes; scattering from inclusions and pores. Matrix anelasticity was thought to contribute in two different ways: through intrinsic anelasticity of matrix minerals and through frictional dissipation due to relative motions at the grain boundaries and across crack surfaces. Attenuation due to intrinsic anelasticity of minerals was assumed to be negligible and to have the wrong frequency dependence, though the idea of frictional sliding of mineral grain boundaries was popular due to works of Walsh (1966) and Mindlin and Deresiewicz (1953). Unlike other proposed mechanisms these frictional dissipation models predicted Q that was independent of frequency as was observed in low-frequency data. However, it was concluded that at low strains ($<10^{-6}$) typical to seismic waves, the behavior of rocks is linear and a nonlinear mechanism such as friction cannot be accepted (Winkler et al. 1979). Besides, these frictional models showed the specific attenuation factor 1/Q to be proportional to strain amplitude while early experiments suggested attenuation independent of strain amplitude (Mavko, 1979); and, finally, it was concluded that frictional losses at small strains were negligible in rock samples, based on calculations by Savage (1969).

However, friction is still considered to be one of the most important sound attenuation mechanisms in marine sediments (Kibblewhite, 1989; Buckingham, 2000). The series of recent observations show the presence of nonlinear effects in rocks at strains as small as 10^{-9} and the lower limit of nonlinearity was not yet observed (Johnson and Rasolofosaon, 1996; Beresnev and Wen, 1996). Over broad ranges of stress, strain, and frequency rocks exhibit nonlinear stress-strain relations, dependence of wave velocity and attenuation on strain amplitude and even the presence of permanent deformation. Indirect evidence for the latter comes from small strain laboratory experiments frequently reporting cusped stress-strain hysteresis loops. The permanent and, importantly, time independent (alternatively termed as irreversible or plastic) deformation in rocks at typical seismic strains was explicitly observed in experiments by Mashinsky (1994) who recorded that irreversible and elastic strains are about equal at strains of the order $10^{-6}-10^{-5}$. Plastic deformation by virtue of its rate independent nature leads to frequency independence of attenuation. Plastic yielding is not expected if a stress-free rock sample is loaded by small seismic strains. However, sediments may be already in a yield state or close to it as a result of complex burial and tectonic loading history. Moreover, rocks are very heterogeneous and these heterogeneities may act as local stress concentrators, so that the actual microscopic stresses around cavities and inclusions may be much higher than the macroscopic stress level. We study attenuation of seismic waves due to local plastic yielding around the cavities in porous media. We are aiming for an understanding of an anomalously high attenuation level frequently inferred for the reservoirs (Korneev et al. 2004; Chapman et al. 2006). Reservoirs may dissipate sound in a similar way to the shallow marine sediments due to either their high porosity or for any other reason.

Model of attenuation due to plastic yielding. In our modeling of attenuation in porous media we follow the effective media theory approach. We consider a single spherical cavity of radius *R* in an incompressible matrix subjected to a confining far-field pressure P^{∞} at the remote boundary (Fig. 1). The cavity wall is subjected to pore pressure *p*. So, the behavior of the solid in general is controlled by the effective pressure $\Delta P = P^{\infty} - p$, which is prescribed to vary in time as a periodic function to imitate a seismic wave coming in and passing away. We assume stresses as positive in tension and pressures positive in compression. If the initial undisturbed effective pressure $\Delta P_0 = P_0^{\infty} - p_0$ is high enough, the plastic region of radius c will develop around the cavity and the initial stress state will be elastoplastic. The distribution of stress, strain and velocity fields in the volume around the cavity during wave propagation we find from the solution of the problem of mechanical equilibrium by methods of mathematical theory of elastoplasticity. Effective properties of the aggregate porous media can be extracted from this solution based on the average theorems. A similar elastoplastic spherical model was previously used by Carroll and Holt (1972) to successfully predict the volumetric response of porous rocks and metals to a hydrostatic pressure within the range of 10-70% porosity.

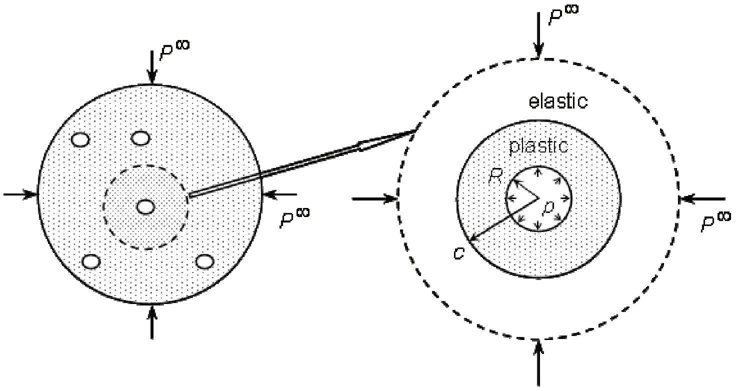

Figure 1. Model of a representative volume element of porous media.

Overall properties of porous aggregate. Average theorems of the effective media theory state that for the representative volume with prescribed boundary tractions the average effective pressure of a porous aggregate is fully defined by the values of pressures at the external and internal boundaries (Nemat-Nasser and Hori, 1999) and, in our case, is equal to ΔP. Porosity φ plays an important role in describing the response of porous media. We define the porosity as a volume fraction of the voids

$$\varphi = V_v / V, \quad V = V_s + V_v$$

where V, V_v and V_s are the total volume and volumes of the cavity and of the solid matrix, respectively. Since the solid matrix is incompressible the porosity equation can be written as $d\varphi/(1-\varphi) = \varphi dV_v/V_v$ .Changes of the void volume V_v are linked to the evolution of the cavity radius R, therefore one can write $d\varphi/(1-\varphi) = 3\varphi dR/R$. The average volumetric strain rate is the surface integral taken over the outer boundary of the representative volume $\overline{\varepsilon_v} = \varphi/V_v \int_{\partial V} v dS$, which due to incompressibility reduces to

$$\overline{\varepsilon_v} = \frac{1}{1-\varphi}\frac{d\varphi}{dt}$$

From this, the average logarithmic volumetric strain is $\overline{e_v} = \ln\left((1-\varphi_0)/(1-\varphi)\right)$. As can be seen, in order to describe the overall properties of porous media one needs to know porosity evolution in the material, which in its turn is totally defined by the radius of the cavity.

Initial stress state around the cavity. According to von Mises yield criterion the plastic region will develop when the effective pressure exceeds the critical value $\Delta P_{cr} = 4Y/3$ with Y being a yield limit of the matrix material. We assume that the initial undisturbed value of the effective pressure is $\Delta P_0 \geq \Delta P_{cr}$. In this case there are two different stress fields in a volume. The first one corresponds to the plastic region $R_0 \leq r \leq c_0$. Here r is a polar radius and subscript 0 designates the initial values of corresponding quantities. Radial stress σ_r and the hoop stress σ_θ in the plastic region are fully determined by an equilibrium equation and von Mises yield criterion:

$$\frac{\partial \sigma_r}{\partial r} + 2\frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (1)$$

$$\sigma_r - \sigma_\theta = 2Y \qquad (2)$$

In the elastic region $r \geq c_0$ one has equilibrium equation (1), Hooke's law $u/r = (\sigma_\theta - \sigma_r)/(6\mu)$ and incompressibility equation $\partial u/\partial r + 2u/r = 0$. These three equations together with condition of stress continuity at the elastoplastic interface define two stress components σ_r, σ_θ and radial displacement *u*.

Loading. A passing seismic wave causes both unloading and further loading. We first consider the latter case resulting in further dissipative plastic flow. Though the solid matrix is incompressible we cannot find the radial velocity *v* independent of stresses since boundary conditions are defined for stresses. We solve the coupled system of incompressibility equation

$$\partial v/\partial r + 2v/r = 0 \qquad (3)$$

and eqs (1), (2) in the plastic region and eqs (1), (3) and incremental version of Hooke's law

$$\frac{v}{r} = \frac{1}{6\mu}\frac{\partial}{\partial t}(\sigma_\theta - \sigma_r) \qquad (4)$$

in the elastic region. At the elastoplastic interface continuity of stresses must be preserved. The solution of these equations gives to us stresses, the radius of the elastoplastic boundary and the value of velocity during the first cycle of loading. Taking into consideration that $v = dR/dt$ at the cavity wall we find the radius of the cavity and porosity equation for loading

$$\frac{d\varphi}{\varphi(1-\varphi)} = -\frac{3}{4}\frac{c^3/R^3}{\mu + Y c^3/R^3}\, d\Delta P$$

where $c = R\exp(\Delta P/(4Y) - 1/3)$.

Unloading. Decreasing effective pressure and elastic unloading of the pore is described by elastic equations (1), (3) and (4) for σ_r, σ_θ and *v*. Porosity increments while unloading are related to the effective pressure increments by

$$\frac{d\varphi}{\varphi(1-\varphi)} = -\frac{3}{4Y\left(1 + 3\ln(R/R_u)\right) + 3(\Delta P - \Delta P_u) + 4\mu}\, d\Delta P$$

where R_u and ΔP_u are values of the cavity radius and effective pressure at the onset of unloading. If the amplitude of the wave is high enough the decrease of the effective pressure may be accompanied by the reverse plastic flow. It will happen if stresses during unloading will reach the critical value $\Delta P_{cr,\, rev\, flow} = \Delta P_u + 4Y/3\left(\ln(1 - 2Y/\mu) - 2\right)$. We do not consider this case here and assume entire unloading is purely elastic. If plastic yielding does not occur during unloading than stresses and velocity evolution during the final reloading part of the cycle are defined by the same equations as during unloading. Since elastic parts of the loading cycle naturally do not contribute to the dissipation we, therefore, give the lower bound estimate for the attenuation.

Results and discussion. As a measure of attenuation in porous media we choose the specific attenuation factor 1/Q, defined as $2\pi/Q = \Delta E/E$, where $\Delta E = -\int_0^{\overline{e_v}} \Delta P\, d\overline{e_v}$ is the amount of energy dissipated per cycle of a harmonic loading and $E = \max(-\Delta P \cdot \overline{e_v})$ is a peak strain energy. Q is further computed as a function of four dimensionless parameters: ratio of initial effective pressure to the yield stress ($\Delta P_0/Y$), strain amplitude *e*, porosity and ratio of the yield stress to the shear modulus (Y/μ). Dependence of Q on Y/μ and porosity is relatively unimportant within the low porosity range of 10-50%. Contouring of Q versus the remaining two parameters is shown on Fig 2a for Y/μ = 33 and 10% porosity. The collapse of this two-dimensional data while allowing for porosity variation in the 10-50% range on a single master curve is presented in Fig 2b. As expected, the attenuation factor 1/Q is strongly strain amplitude dependent at large strains ($>10^{-5}$) and is indeed negligible if the initial effective pressure is small. However, at larger initial effective pressures, large values of attenuation are predicted while preserving strain amplitude independence for the small strain amplitudes ($<10^{-5}$) in agreement with experimental observations and intuitive expectations.

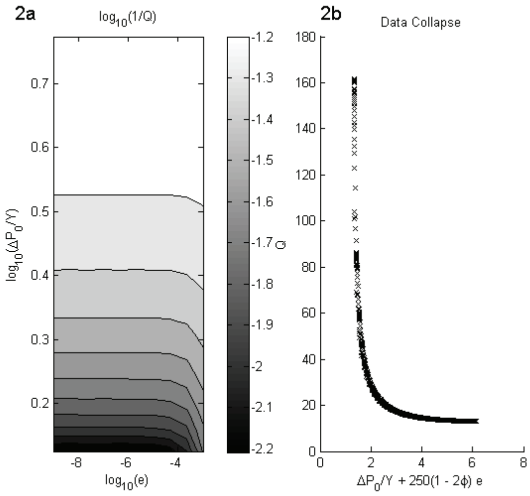

Figure 2. Model predictions and data collapse for quality factor Q.

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