Surface wave spectral ratio inversion for shear velocity

E. Muyzert (Schlumberger Cambridge Research)

SUMMARY

Introduction

Phase and group velocities are the most commonly derived measurements from surface waves. They are known to be robust measurements and the theory and inversion of surface wave phase and group velocities are well developed and successful. They are commonly used to derive shear velocity profiles and attempts have been made to invert for P-wave, density and anisotropy parameters as well. The drawback of these phase and group velocity methods is that an active source is typically required and that the phase-velocity methods also require a receiver array. As such they do not provide a truly local measure of the ground.

We show that surface wave amplitude information can provide local measurements. It is assumed that three component measurements of the ground motion are available. The spectral ratio of the horizontal energy over the vertical component (H/V) energy is one such measure amplitude measurement. It was originally developed for site characterization and determining the local shear velocity for earthquake hazard studies (Nakamura 1989). The benefit of the H/V spectral ratio is that it is independent of the source amplitude and propagation effects. Another benefit is that both noise and signal records can be used. Here I will investigate various aspects of the Horizontal over Vertical spectral ratio and propose an alternative measure, the Vertical of Total (V/T) spectral ratio that is more suited for inversion.

Horizontal over Vertical spectral ratio

The H/V spectral ratio method uses ground-roll or interface wave data from three component recordings of the ground motion. The data may be from an active seismic source like an earthquake or explosives, or ambient noise records that are dominated by ambient ground-roll may be used. The horizontal component data is rotated towards the inline direction which is used for further processing. The crossline component data that will contain mainly scattered ground-roll and Love waves is not used. In the case of ambient noise or heavily scattered surface waves the wavefront direction is not known in which case the squared sum of the horizontal components has been taken. These data may be still contaminated by Love waves and other energy.

The spectra of the horizontal trace is divided by the spectrum of the vertical trace.

$$H / V = H(f) / V(f)$$

resulting in the Horizontal over Vertical (H/V) spectral ratio. It is assumed that the input waveforms are dominated by a particular wavetype, usually surface waves, Rayleigh waves in particular, with a vertical inline particular motion. The amplitude of the H/V spectral ratio depends only on the local Rayleigh wave eigen function (Fäh, 2001). Other modes such as Love waves are ignored. Next one picks the peak in H/V spectral ratio. The frequency at which this peak occurs is where the frequency of the vertical component is zero. A simple model of a layer over a halfspace is used to explain this zero vertical component amplitude. The depth of the model is at a quarter of a wavelength, thus a range of s-velocities and depths can agree to this model. While this has proven to be a useful method it uses only a small part of the information in the spectrum is used and this requires considerable manual input.

Spectral ratio inversion

While the H/V spectral ratio has proven to be a useful measurement we will here discuss some improvements. Modeling has shown that interpretation of the H/V spectral ratio method is not straightforward for realistic seabeds. Figure 1 shows three different near surface seabed velocity models, the first is the empirical relation by Hamilton (1976), the second model is like Hamilton's but with a low velocity layer in the top 30 m. The third model has this step replaced a gradient. Figure 2 shows the H/V spectral ratio calculated for these three models. While clear peaks are observed in the H/V spectral ratio for the step and gradient model that both have a well defined low velocity zone, the spectral ratio for the Hamilton model does not have such a well defined peak. Another issue with the H/V spectral ratio is that only one data point is used, the peak frequency.

In our new method an inversion of the spectral ratio for shear velocity is carried out. We use the full bandwidth over which the spectral ratio data is available. This would result in better depth resolution than the tradition H/V spectral ratio method. In addition not the H/V spectral ratio is used but Vertical over Total spectral. The total spectrum is the squared sum of the horizontal and vertical components.

$$V/T = V(f) / \sqrt{H(f)^2 + V(f)^2}$$

We did explore various other spectral ratios but the V/T spectral ratio was preferred because of its inherent protection against a division by zero and its corresponding asymptote. It is a relatively well behaved and by definition its amplitude is between 0 and 1, see Figure 2. It does not have sharp peaks that would dominate an inversion procedure.

In the inversion procedure the misfit between the observed V/T spectral ratio and a synthetic V/T spectral ratio calculated for a realistic model is measured by the L^2 norm and minimized using an iterative least square linear procedure. In addition some model smoothness constrains were added to the misfit function. It was found that the choice of the starting model was important in order to find the global minimum of the misfit function. As the calculation of synthetic H/T spectra is fast global methods such as genetic algorithms may be used to solve the inverse problem.

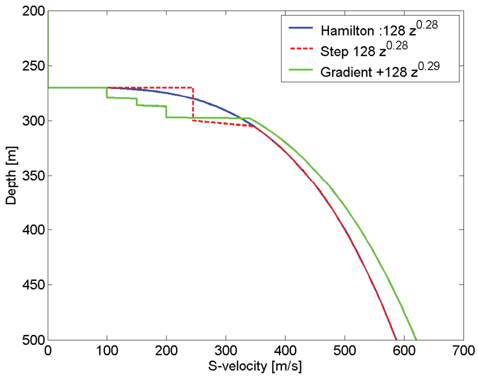

***Figure 1** Shear wave velocity models.*

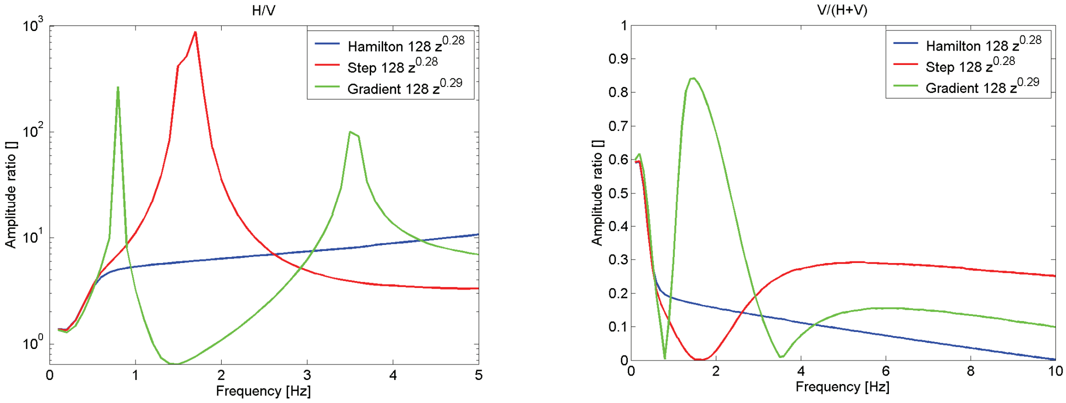

***Figure 2** The H/V (left) and V/T (right) spectral ratios calculated for the velocity models displayed in Figure 1. Note that logarithmic scaling has been applied to the H/V spectral ratio plot.*

A seabed example

The V/T spectral ratio method was applied to a seabed ambient noise data-set. A near surface shear velocity model derived from Scholte wave phase-velocities along the receiver area was available and used as starting model (Muyzert, 2007). The data was filtered such that non Scholte wave energy was removed and V/T spectral ratios were calculated for all 448 available receivers and summed, excluding bad traces. Initial modeling showed that the Hamilton model does not explain the observed V/T spectral ratio, see Figure 3. Synthetics calculated for the model derived from phase-velocity data were closer to the observed V/T spectral ratio and used as starting model in the inversion. After a V/T inversion a good fit was obtained. The resulting shear velocity model is shown in Figure 4 and shows that only relatively small changes to the model were required to get the data fit. The obtained velocity model was used as a starting model for the inversion of the spectral ratios for each of the 448 individual receivers.

Land data

The same procedure may be applied to land seismic data with an active source. The data is best selected such that it is dominated by ground-roll and reflections are minimized through processing. In addition one can attempt to minimize the higher modes that have a higher velocity than the fundamental mode ground-roll. It should be noted that it is not clear what the benefit is of using ambient noise land data if shot generated ground-roll with a much higher Signal to Noise ratio is also available. A modeling study has been carried out that investigates the impact of the expected velocities and Vp/Vs ratios on the different spectral ratios.

Conclusions

A novel approach to the inversion of Vertical over Total spectral ratio has been presented. The method assumes that the data is dominated by a single mode of surface waves. Is has been shown that the V/T spectral ratio uses a larger part of the spectrum that the H/V spectral ratio method, providing more depth resolution. Another benefit is that it is a local measurement. Although a linearized inversion may be carried, alternatively more global searches may be carried out such as genetic algorithms. The method can be applied to Scholte waves present in seabed recordings and Rayleigh waves recorded in three component land data.

References

Fäh, D., Kind, F. and D. Giardini, 2001, A theoretical investigation of average H/V ratios: Geophys. J. Int., **145**, 535-549.

Hamilton, E.L. 1976. Shear-wave velocity versus depth in marine sediments: a review, Geophysics, 41, 985-996.

Muyzert, E., 2007. Seabed property estimation from ambient noise recordings, part 2: Scholte wave spectral ratio inversion, Geophysics 72, U47.

Nakamura, Y., 1989, A method for the dynamic characteristics estimation of subsurface using microtremor on the ground surface: QR. of RTRI, **30**, 25-33.

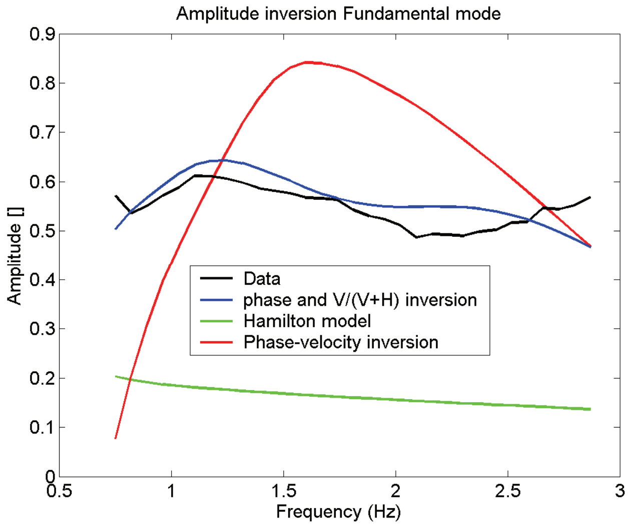

Figure 3 *The observed Vertical over total spectral ratio for a seabed data-set (black), the modeled for Hamilton's model (green), and the spectral ratios obtained by a phase-velocity and V/T spectral ratio inversion.*

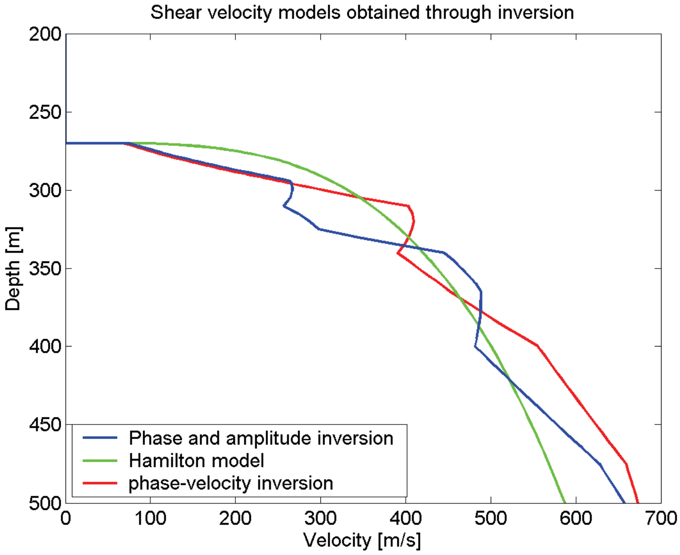

Figure 4 *Shear wave velocity models obtained in the joint inversion of phase-velocity and V/(V+H) spectral ratios.*