Attenuation and dispersion of compressional elastic waves due to wave-induced flow in random porous media

Tobias M. Müller & Boris Gurevich
Curtin University, Perth

February 3, 2005

We develop a model for wave attenuation and dispersion due to wave-induced fluid flow in 3-dimensional randomly inhomogeneous poroelastic media. Using the method of statistical smoothing applied to Biot's equations of poroelasticity, we study the dynamic characteristics of the compressional wave mode. Attenuation and dispersion of the wave depend on linear combinations of the spatial correlations of the fluctuating poroelastic parameters. The observed frequency dependence is typical for a relaxation phenomenon. The results show that even small spatial fluctuations in the poroelastic parameters produce significant attenuation and dispersion. Our approach can be applied to model the effect of partial fluid saturation and its relation to medium heterogeneity.

1 INTRODUCTION

One major cause of elastic wave attenuation in porous fluid-saturated materials is viscous dissipation due to the flow of the pore fluid induced by the passing wave. Wave-induced fluid flow occurs as a passing wave creates local pressure gradients within the fluid phase and the resulting fluid flow is accompanied with internal friction until the pore pressure is equilibrated. The fluid flow can take place on various length scales: for example from compliant fractures into the equant pores (so-called squirt flow (Mavko and Nur 1975)), or between mesoscopic heterogeneities like fluid patches in partially saturated rocks (Murphy 1982; Knight et al. 1998).

The first models of attenuation due to wave-induced flow considered flow caused by a regular assemblage of inhomogeneities of ideal shape such as two concentric spheres or flat slabs (White et al. 1975; Dutta and Ode 1979). A general theory of wave propagation in heterogeneous porous media using the double-porosity approach was recently developed by Pride and Berryman (2003). Although mathematical formulations in the theories of Pride and Berryman allow for arbitrary geometry of the inhomogeneities, closed-form expressions for attenuation and phase velocity can only be obtained for a regular system of simple geometrical shapes such as concentric spheres.

In real porous materials heterogeneities are more likely to be spatially distributed in a random fashion. Therefore, it is desirable to model wave induced flow using the theory of waves in random media. For 1-dimensional porous media Gurevich and Lopatnikov (1995) showed that elastic wave attenuation in a randomly layered porous medium differs significantly from attenuation in periodically layered porous media (White et al. 1975; Norris 1993). This suggests that the effects of 3–dimensional inhomogeneities on elastic wave wave attenuation and dispersion in porous media may also be different for random and periodic spatial configurations. The purpose of this paper is to analyze the effect of wave-induced fluid flow on the dynamic characteristics of coherent elastic waves propagating in a porous medium whose properties are continuous random functions of position. We restrict our analysis to the case of mesoscopic inhomogeneities, that is, inhomogeneities whose characteristic size a is much larger than typical size of pores or grains a_{pore} but, at the same time, much smaller than the wavelength of the propagating elastic wave, λ:

$$a_{\text{pore}} \ll a \ll \lambda \,. \tag{1}$$

In other words, we ignore pore-scale heterogeneities, which allows the application of Biot's equations for a poroelastic continuum (Biot 1962). By using a Green's function approach these partial differential equations transform into a system of integral equations. This latter system is solved by means of the method of statistical smoothing which is widely used in problems of electromagnetic, acoustic and elastic wave propagation (Karal and Keller 1964). More precisely, we employ a first-order statistical smoothing approximation. We analyze the conversion scattering from fast compressional ($P-$) waves into Biot's slow wave in 3-D randomly inhomogeneous porous media. Biot's slow wave is a highly dissipative wave mode. Therefore, the use of the first-order statistical smoothing approximation to the conversion scattering problem in Biot's equations of poroelasticity quantifies the dissipation of wave field energy due to energy transfer from the coherent component of the fast P-wave into the dissipative slow P-wave mode. This is different from the usual application of the method of smoothing to energy conserving wave systems, where an apparent dissipation (so-called scattering attenuation) results from the energy transfer from the coherent component of the wave field into the incoherent component.

The outline of this paper is as follows. First, we apply the method of Green's functions to Biot's equations of poroelasticity in order to represent the wavefield due to point source excitation (section 2). Next, we derive an integral wavefield representation for the case when the coefficients in Biot's equations exhibit a randomly fluctuating component. This wavefield representation is then converted into an integral equation for Green's function for the inhomogeneous medium. In section 3 we apply the method of statistical smoothing to compute an approximation for the mean of the Green's function. From this Green's function we derive an explicit expression for the effective, complex P-wavenumber which accounts for the conversion scattering from P- into slow P-waves. From the effective wavenumber we derive expressions for attenuation and velocity dispersion. Using the method of smoothing we also obtain an expression for an effective slow P-wavenumber from which we identify an effective permeability (section 3.2). Then, we investigate how permeability fluctuations influence the attenuation of the fast compressional wave mode. The conclusions are presented in section 4.

2 FORMULATION OF THE PROBLEM

In order to study dynamic effects of seismic waves in porous media, we base our analysis on Biot's (1962) equation of poroelasticity. We restrict our analysis to low frequencies. That means we assume that the frequency ω is much smaller than the critical Biot frequency $\omega_B = \phi\eta/\kappa\rho_f$:

$$\omega \ll \omega_B\,, \tag{2}$$

where ϕ and κ denote the porosity and permeability of the composite material, while ρ_f and η are the density and viscosity of the pore fluid. Condition (2) implies that the standard Biot's visco-inertial attenuation and dispersion (so-called Biot's global flow mechanism) is neglected. Using index notation – summation over repeated indices is assumed and partial derivatives are denoted as $_{,i}$ or ∂_i – we can write the low-frequency version of Biot's equations in matrix form (the time-harmonic dependency $\exp(-i\omega t)$ is omitted)

$$\begin{bmatrix} L_{ik}^{(1)} & L_{ik}^{(2)} \\ L_{ik}^{(3)} & L_{ik}^{(4)} \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix} = \mathbf{0}\,, \tag{3}$$

where we defined the linear differential operators as follows

$$L_{ik}^{(1)} = \rho\omega^2\delta_{ik} + \partial_j G[\delta_{jk}\partial_i + \delta_{ik}\partial_j - 2\delta_{ij}\partial_k] + +\partial_i H \partial_k \tag{4}$$

$$L_{ik}^{(2)} = \rho_f\omega^2\delta_{ik} + \partial_i C\partial_k \tag{5}$$

$$L_{ik}^{(3)} = L_{ik}^{(2)} \tag{6}$$

$$L_{ik}^{(4)} = q\omega^2\delta_{ik} + \partial_i M\partial_k\,. \tag{7}$$

Here the u_i and w_i are components the solid and relative fluid displacement vectors, respectively. The latter is defined as $w_i = \phi(U_i - u_i)$ with fluid displacement vector U_i. The material parameters are given by the bulk density ρ and $\rho^\star = i/\omega\kappa_0$ with the conductivity $\kappa_0 = \kappa/\eta$. If the density of the solid is ρ_g, the bulk density can be expressed as $\rho = \phi\rho_f + (1-\phi)\rho_g$. Here G is the shear modulus, $H = P_d + \alpha^2 M$ the saturated P-wave modulus, and P_d the P-wave modulus of the dry frame. The poroelastic parameters α, M and C are defined in the standard way as $\alpha = 1 - K_d/K_g$, $M = [(\alpha-\phi)/K_g + \phi/K_f]^{-1}$ and $C = \alpha M$. The quantities K_g, K_d, K_f denote the bulk moduli of the solid phase , the dry frame, and the pore fluid, respectively.

In randomly inhomogeneous porous media, all poroelastic parameters can be presented as random fields $X(\mathbf{r})$. We assume that these random fields are the sum of a constant background value, $\bar{X}(\mathbf{r})$ and a fluctuating part, $\tilde{X}(\mathbf{r})$, so that $X = \bar{X} + \tilde{X}$. The average over the ensemble of the realizations (denoted by $\langle\cdot\rangle$) of $\tilde{X}$ is zero: $\langle\tilde{X}\rangle = 0$. Consequently, the differential operators L_{ik} can be also written as $L_{ik} = \bar{L}_{ik} + \tilde{L}_{ik}$, where the perturbing operator $\tilde{L}_{ik}$ satisfies $\langle\tilde{L}_{ik}\rangle = 0$. In the most general case, the perturbing operators contain fluctuations of all poroelastic moduli and densities. In analogy to the elastic case ((Gubernatis et al. 1977)) the perturbed system (3) can be converted into an integral equation of the form

$$\begin{bmatrix} u_i \\ w_i \end{bmatrix} = \begin{bmatrix} u_i^0 \\ w_i^0 \end{bmatrix} +$$

$$+ \int_V dV \begin{bmatrix} G_{ij}^F & G_{ij}^f \\ G_{ij}^f & G_{ij}^w \end{bmatrix} \begin{bmatrix} \tilde{L}_{jk}^1 & \tilde{L}_{jk}^2 \\ \tilde{L}_{jk}^3 & \tilde{L}_{jk}^4 \end{bmatrix} \begin{bmatrix} u_k \\ w_k \end{bmatrix} \quad (8)$$

where the total wavefields **u** and **w** are composed of wavefields propagating in the homogeneous background medium ($\mathbf{u}^0$ and $\mathbf{w}^0$) and scattered wavefields (represented by the second term). The scattered wavefields vanish if there are no fluctuations in the medium parameters. The integration volume encompasses the inhomogeneous part of the medium. The Green tensors G_{ij} of a poroelastic continuum in 3-D space are known (Pride and Haartsen 1996). In the low-frequency regime (condition 2) these tensors can be simplified to

$$G_{ij}^F = \frac{1}{4\pi\rho\omega^2}\left(\left[k_s^2\delta_{ij} + \partial_i\partial_j\right]\frac{e^{\mathrm{i}k_sR}}{R} - \partial_i\partial_j\frac{e^{\mathrm{i}k_pR}}{R}\right)$$

$$-\frac{C^2}{H^2}\frac{1}{4\pi q\omega^2}\partial_i\partial_j\frac{e^{\mathrm{i}k_{ps}R}}{R} \quad (9)$$

$$G_{ij}^f = \frac{C}{H}\frac{1}{4\pi q\omega^2}\partial_i\partial_j\frac{e^{\mathrm{i}k_{ps}R}}{R} \quad (10)$$

$$G_{ij}^w = -\frac{1}{4\pi q\omega^2}\partial_i\partial_j\frac{e^{\mathrm{i}k_{ps}R}}{R}, \quad (11)$$

where $R = |\mathbf{r} - \mathbf{r}_0|$. In homogeneous and isotropic media the Green's tensors only depend on R. In the low-frequency version of Biot's equations the wavenumbers of fast P-, S-, and slow P-waves are defined as

$$k_p = \omega\sqrt{\frac{\rho}{H}} \qquad k_s = \omega\sqrt{\frac{\rho}{G}} \qquad k_{ps} = \omega\sqrt{\frac{\rho^\star}{N}}, \quad (12)$$

where $N = MP_d/H$. Since equation (8) must be also true for the Green tensors G_{ij}, we obtain

$$\mathbf{G} = \mathbf{G}^0 + \int \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}, \quad (13)$$

where all these quantities represent 2×2 matrices whose elements are tensors of rank two.

3 METHOD OF STATISTICAL SMOOTHING

We will now analyze equation (13) using a statistical approach. Since the matrix of perturbing operators $\tilde{\mathbf{L}}$ in equation (13) contains fluctuating medium parameters, the resulting matrix of Green's tensors also contains randomly fluctuating elements. Because individual realizations of the random wavefields are never known, it is expedient to analyze the statistical moments of **G**. Solving equation (13) by iteration we obtain

$$\mathbf{G} = \mathbf{G}^0 + \int \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0 + \int\int \mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0 + \int\int\int \ldots . \quad (14)$$

Averaging this equation by the ensemble of realizations and regrouping the scattering terms yields

$$\bar{\mathbf{G}} = \mathbf{G}^0 + \int\int \mathbf{G}^0\mathbf{Q}\bar{\mathbf{G}}, \quad (15)$$

where $\bar{\mathbf{G}} = \langle\mathbf{G}\rangle$ is the matrix of mean Green's tensors, and **Q** is the matrix operator defined as

$$\mathbf{Q} = \begin{bmatrix} Q_{ik}^{(1)} & Q_{ik}^{(2)} \\ Q_{ik}^{(3)} & Q_{ik}^{(4)} \end{bmatrix}$$

$$= \left\langle \tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}} + \int \tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}} + \int \ldots \right\rangle . \quad (16)$$

Operator **Q** given by equation (16) corresponds to the kernel-of-mass operator in the acoustic formulation (Rytov et al. 1989). The linear integral equation in $\bar{\mathbf{G}}$ (equation (15)) is the poroelastic analog of the Dyson equation. It is not possible to obtain an exact solution of equation (15). A first-order statistical smoothing consists in the first-order truncation of the infinite series expression for the operator **Q**. Then, we obtain the following approximation for the mean Green's tensor:

$$\bar{\mathbf{G}} = \mathbf{G}^0 + \int\int \mathbf{G}^0\left\langle\tilde{\mathbf{L}}\mathbf{G}^0\tilde{\mathbf{L}}\right\rangle\bar{\mathbf{G}} \quad (17)$$

or

$$\bar{\mathbf{G}} = \mathbf{G}^0 + \int\int \mathbf{G}^0\mathbf{Q}^B\bar{\mathbf{G}}. \quad (18)$$

The truncation of the series (16) implies that the first-order statistical smoothing is valid when $|\varepsilon_X| \ll 1$, i.e., when the absolute value of the relative fluctuations of X is a small parameter.

3.1 FAST COMPRESSIONAL WAVES

The problem can be further simplified if we note that we are not interested in the full Green tensor itself but in an effective P-wavenumber only. To do so, we consider an incoming, plane P-wave propagating in x_3-direction (i.e. only the displacement component u_3 is nonzero). The resulting coherent P-wave in the inhomogeneous medium will also propagate in x_3-direction. Then we obtain for the effective wavenumber

$$\bar{k}_p \approx k_p\left(1 + \frac{4\pi^3}{\rho\omega^2}q_{33}^{(1)}\right). \quad (19)$$

The remaining problem is the evaluation of $q_{33}^{(1)}$ in (19) which is the spatial Fourier transform of $Q_{33}^{(1)}$. In explicit form, from the first term in the expansion of **Q** as given by equation (16) we obtain

$$\begin{aligned} Q_{ik}^{(1)} &= \Big\langle \tilde{L}_{ij}^{(1)}(\mathbf{r}')G_{jl}^{F}(\mathbf{r}'-\mathbf{r}'')\tilde{L}_{lk}^{(1)}(\mathbf{r}'') + \\ & 2\tilde{L}_{ij}^{(1)}(\mathbf{r}')G_{jl}^{f}(\mathbf{r}'-\mathbf{r}'')\tilde{L}_{lk}^{(2)}(\mathbf{r}'') + \\ & \tilde{L}_{ij}^{(2)}(\mathbf{r}')G_{jl}^{w}(\mathbf{r}'-\mathbf{r}'')\tilde{L}_{lk}^{(2)}(\mathbf{r}'')\Big\rangle , \end{aligned} \quad (20)$$

where for statistically homogeneous random media both Q_{ik} and G_{ik} depend only on the difference vector $\mathbf{r}' - \mathbf{r}''$. To proceed with the analysis we neglect fluctuations of the densities ρ, ρ_f, and $\rho^\star$. This is possible because of the restriction to low frequencies (Gelinsky et al. 1998). Using these simplified perturbation operators we evaluate $q_{33}^{(1)}$ in (19) and obtain for the effective P-wavenumber

$$\bar{k}_p = k_p\Big(1 + \Delta_2 + \Delta_1 k_{ps}^2 \int_0^\infty rB(r)\exp[\mathrm{i}k_{ps}r]\,dr\Big)\,, \quad (21)$$

where Δ_1 and Δ_2 are the dimensionless coefficients

$$\begin{aligned} \Delta_1 &= \frac{\alpha^2 M}{2P_d}\Big(\sigma_{HH}^2 - 2\sigma_{HC}^2 + \sigma_{CC}^2 + \frac{32}{15}\frac{G^2}{H^2}\sigma_{GG}^2 \\ & -\frac{8}{3}\frac{G}{H}\sigma_{HG}^2 + \frac{8}{3}\frac{G}{H}\sigma_{GC}^2\Big) \end{aligned} \quad (22)$$

$$\begin{aligned} \Delta_2 &= \Delta_1 + \frac{1}{2}\sigma_{HH}^2 - \frac{4}{3}\frac{G}{H}\sigma_{HG}^2 + \\ & \left(\frac{4G}{H}+1\right)\frac{4}{15}\frac{G}{H}\sigma_{GG}^2 . \end{aligned} \quad (23)$$

Equation (21) was obtained by assuming that the parameters H, G, and C have a random component. The correlation properties of the random inhomogeneities are characterized by the normalized correlation function $B(r)$ which for the three random functions H, G and C assumes the same functional form. The variances of the relative fluctuations are denoted as σ_{HH}^2, σ_{GG}^2 and σ_{CC}^2. The cross-variance of the relative fluctuations are σ_{HG}^2, σ_{HC}^2 and σ_{GC}^2.

It is important to note that the above expression describes only the process of conversion scattering from fast to slow P-waves. The contribution of purely elastic scattering is not contained in equation (21). Therefore, from the complex P-wavenumber (21) we can extract the attenuation and dispersion characteristics due to wave-induced flow only. Equation (21) for the effective wave number enables us to derive expressions for the attenuation and dispersion due to the presence of mesoscopic inhomogeneities. By definition, the real part of $\bar{k}_p$ yields the phase velocity

$$\begin{aligned} v(\omega) &= v_0\Big[1 - \Delta_2 + 2\Delta_1\bar{k}^2 \\ & \int_0^\infty rB(r)\exp[-\bar{k}r]\sin(\bar{k}r)\,dr\Big] \end{aligned} ,(24)$$

where v_0 is the constant background P-wave velocity defined as $v_0 = \sqrt{H/\rho}$ (ρ is the bulk density) and $\bar{k}$ denotes the real part of the slow P-wave number k_{ps}. The imaginary part of the wave number yields the attenuation coefficient γ which in turn yields the reciprocal quality factor Q^{-1}:

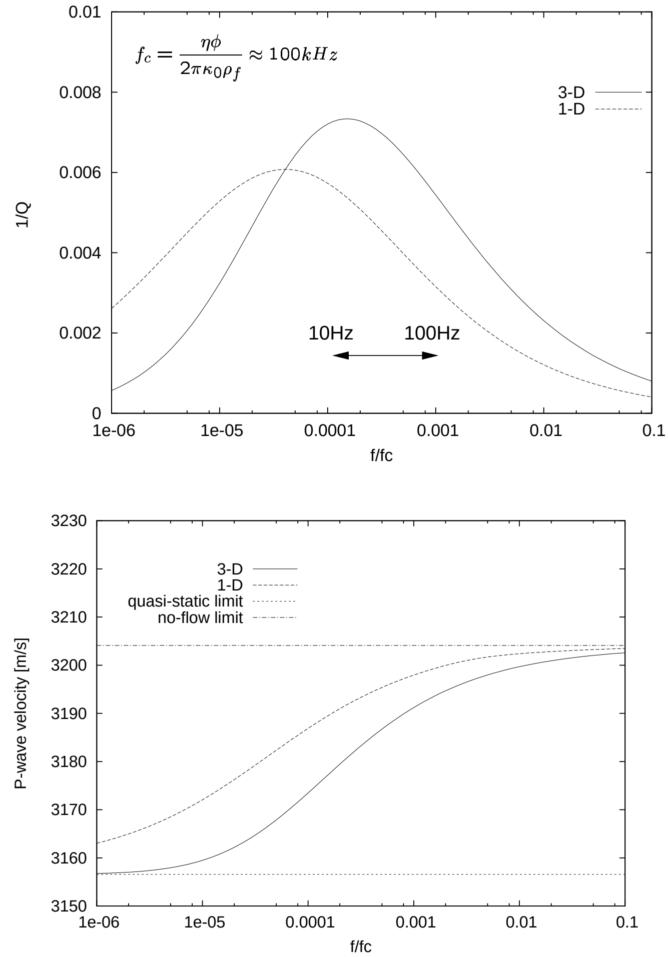
$$Q^{-1}(\omega) = 4\Delta_1\bar{k}^2\int_0^\infty rB(r)\exp[-\bar{k}r]\cos(\bar{k}r)\,dr\,. \quad (25)$$

From equations (24) and (25) the meaning of the coefficients (22) and (23) becomes clear. The attenuation Q^{-1} and the frequency-dependent part of v are proportional to Δ_1. Thus Δ_1 is the measure of the magnitude of attenuation and velocity dispersion, that is, the dynamic effects. In contrast, Δ_2 produces a frequency-independent velocity shift in (24).

To illustrate the above results let us consider an example. For a typical porous sandstone with porosity $\phi = 17\%$ and permeability $\kappa = 250\mathrm{mD}$ we use the following background parameters: $P_d = 16.5\mathrm{GPa}$, $\alpha = 0.89$, and $M = 10.4\mathrm{GPa}$. The pore fluid is water with viscosity $\eta = 0.001\mathrm{Pa\ s}$. The correlation function is of the form $B_n(r) = \sigma_n^2\exp[-|r/a|]$, where σ_n^2 is the variance of the fluctuations and a the correlation length. There are fluctuations in the fluid bulk modulus K_f with variances $\sigma_{K_fK_f}^2 = 0.2$, The correlation length $a = 10\mathrm{cm}$. For such a model we obtain the frequency dependence of attenuation and phase velocity as shown in Figures 1a and 1b, respectively. The frequency is normalized by Biot's critical frequency, which in this example is $f_c \approx 100\mathrm{kHz}$.

3.2 SLOW COMPRESSIONAL WAVES

Using the method of statistical smoothing it is also possible to derive from equation (18) an expression for an effective slow P-wavenumber. The slow P-wave in a homogeneous porous medium is a highly dissipative wave mode. In inhomogeneous porous medium the effective slow P-wave will be additionally attenuated and dispersed due to interaction with inhomogeneities. Instead of analyzing the properties of this effective slow P-wavenumber we are interested in the determination of an effective conductivity (permeability). Assuming that the effective slow P-wavenumber k_{ps}^* is of the form $k_{ps}^* = \sqrt{i\omega/(\kappa^* N)}$



a)
b)

Figure 1: Attenuation in terms of Q^{-1}(a) and velocity dispersion (b) as a function of normalized frequency for a model with fluctuations in the parameter K_f with $\sigma^2_{K_f K_f} = 0.2$ and $a = 10$cm. For the same parameters the result of the 1-D poroelastic extension of the ODA theory (Gelinsky et al. 1998) is also shown (dashed curve).

involving the effective conductivity, we obtain an explicit equation for κ^*. For statistically isotropic random media we find

$$\kappa^* = \kappa_0 \left(1 - \frac{\sigma^2}{3} + \frac{8}{3}\sigma^2_{\kappa\kappa}\bar{k}^2 \times \right.$$
$$\left. \times \int_0^\infty rB(r)\exp(-\bar{k}r)\sin(\bar{k}r)dr\right) ,(26)$$

where $\sigma^2_{\kappa\kappa}$ denotes the variance of the reciprocal conductivity fluctuations. Equation (26) provides a frequency-dependent expression for the effective conductivity. The integral in equation (26) involving the spatial correlation function shows the non-local character of the effective conductivity. In the low-frequency limit we obtain from (26):

$$\kappa^* = \kappa_0 \left(1 - \frac{\sigma^2}{3}\right) . \qquad (27)$$

It is interesting to note that this low-frequency limit was also obtained by Keller (2001) using the method

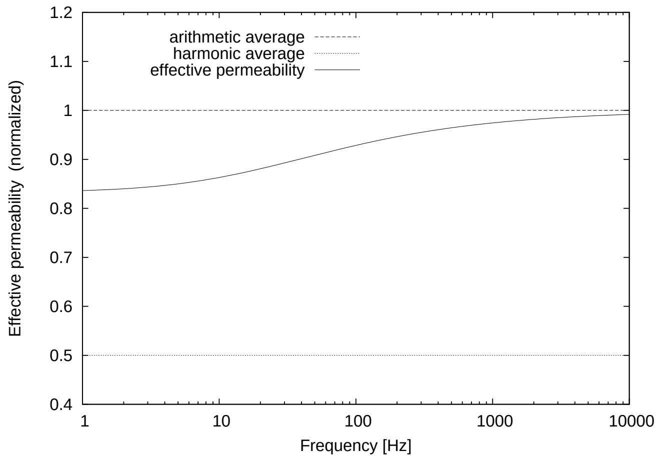


Figure 2: Frequency dependence of permeability (normalized by the background permeability κ_b) in random porous media according to equation (26). It can be observed that the effective permeability is always larger than the harmonically averaged permeability and smaller than the arithmetically averaged permeability (in the high-frequency limit the effective permeability and the arithmetic average coincide).

of smoothing directly applied to Darcy's law. It is well-known that the factor $1/3$ in equation (27) reflects the assumption of statistical isotropy in 3-D random media and is typical for effective conductivity analysis in weakly inhomogeneous structures (Beran 1968). In the high-frequency limit equation (26) yields $\kappa^* = \kappa_0$ i.e., the arithmetic average of the conductivity values. Figure 2 depicts the frequency dependence of the effective conductivity according to equation (26) for an exponential correlation function $B(r) = \exp(-|r|/a)$ with variance $\sigma^2_{\kappa\kappa} = 0.5$ and $a = 0.15$m (the background permeability is $\kappa = 250$mD, viscosity $\eta = 0.001$Pas, $N = 6.9$GPa).

In a next step we analyze the effect of permeability fluctuations on fast P-wave attenuation. Equation (25) – based on first-order perturbation approximation – does not account for conductivity fluctuations and only involves a constant (background permeability). However we can do better: A 'second-order' approximation can be obtained by replacing the background slow P-wavenumber in equation (25) by its first-order approximation. Moreover, neglecting the cross-correlations between the fluctuations of elastic moduli and conductivity, and assuming that the variance of the conductivity fluctuations is larger than those of the elastic moduli, we can construct the 'second-order' approximation by simply replacing the background conductivity by the effective conductivity (equation 26). Figure 3 shows the reciprocal quality factor according to equation (25) with $\Delta_1 = 0.027$ (all other parameters are the same as in Figure 2). If a constant background conductivity value is used we obtain the dashed curve. Using equation (25) in conjunction with the effective conductivity we obtain the

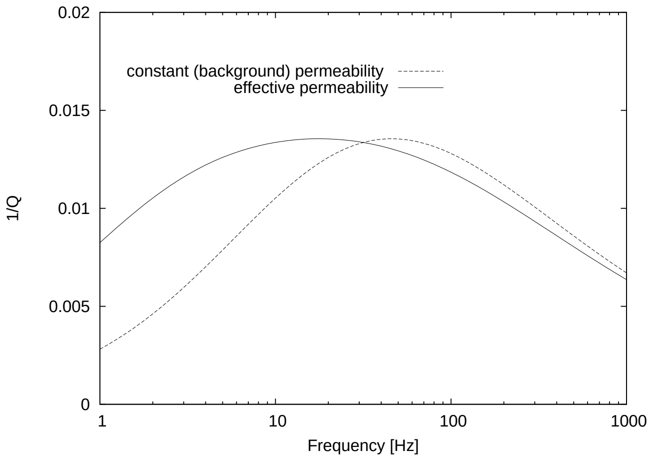


Figure 3: Frequency dependence of inverse quality factor accounting for dissipation due to wave-induced flow. In the case of a constant permeability medium attenuation according to equation (25) is given by the dashed curve. Incorporation of the dynamic permeability model in the expression for attenuation produces the solid curve. It can be observed that due to the presence of permeability fluctuations the maximal attenuation occurs at lower frequencies.

solid curve (here we used $\sigma^2_{\kappa\kappa} = 1$). Two effects can be observed in Figure 3: First, attenuation of the fast $P-$wave reaches maximum at lower frequencies if the dynamic conductivity (26) is used (with respect to the attenuation observed for the constant background permeability case). Second, incorporation of the dynamic conductivity in equation (25) results in a broadening of the attenuation peak.

4 CONCLUSIONS

Based on the method of statistical smoothing we proposed a model for frequency-dependent attenuation and dispersion in 3-D randomly inhomogeneous porous materials accounting for the effect of wave-induced fluid flow. In our approach the dynamic characteristics depend on the correlation properties of the medium fluctuations. Explicit results for $Q^{-1}(\omega)$ and $v(\omega)$ can be obtained for certain correlation functions. Even weak fluctuations of the poroelastic parameters can result in significant attenuation and dispersion. These signatures are different in 1-D and 3-D inhomogeneous structures. We also derive a model of frequency-dependent permeability using the theoretical framework of statistical smoothing. This enables us to infer the impact of permeability fluctuations on the dynamic characteristics of the propagating wave mode. The results indicate that in the presence of large permeability fluctuations the signatures of wave-induced fluid flow on the attenuation of the compressional wave can be observed in a broader frequency range than previously thought.

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