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On the emergence of the Green's function in the correlations of a diffuse field: pulse-echo using thermal phonons

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Abstract

It is shown that a diffuse field is not devoid of phase information, but has a correlation function equal to the Green's function. More specifically, the cross-correlation between diffuse signals in two transducers is very nearly equal to the direct response of one transducer to an impulse applied to the other. This is true whether the diffuse field is one that was created by a distant source, or (if the detectors are sufficiently sensitive) created by thermal fluctuations in the specimen. Here we outline and review proofs, and laboratory demonstrations, from three recent archival publications. © 2002 Elsevier Science B.V. All rights reserved.

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Recent years have seen a substantial and growing literature on diffuse ultrasonic elastic waves. These are waves which have multiply scattered, or multiply reflected, sufficiently often and sufficiently irregularly that detailed modeling of their propagation becomes impossible. Long-lived waveforms that can ensue from transient loads in such bodies have the appearance of non-stationary random noise, and signal processing methods developed for treating such noise are often employed [1]. The concepts were first developed for room acoustics [2]. But applications have arisen in non-destructive testing, where internal friction [3], single back-scattering from microstructure [4], and multiply scattered grain noise [5], are evaluated. The concepts appear to be relevant for seismic waves [6] also. Diffuse elastic waves that arise in simple reverberant bodies are providing a laboratory analog for quantum chaos and mesoscopic electronics [7]. Throughout this work it has been routinely presumed that a diffuse field, though it may have explored a large volume, contains little detailed information. Work has mostly been confined to extracting specimen properties by analysis of gross features in the evolution of energy density.

Here we report that a diffuse field in fact contains, in its fine-structure correlations, all the information also contained in conventional ultrasonics. A "diffuse" field is not "incoherent." We also report that the perfectly diffuse fields formed by thermal fluctuations in the elastic wave field (thermal phonons as it were) are ideal for extracting that information. This remarkable result has been reported in three recent publications [8–10], which are reviewed here.

A proof of this correlation is not difficult to sketch. An elastic wave field in a finite body may be expressed as a normal mode expansion

$$\mathbf{u}(\mathbf{x},t) = \mathrm{Re}\sum_{n=1}^{n=\infty} a_n \mathbf{u}^{(n)}(\mathbf{x}) \exp\{i\omega_n t\} \tag{1}$$

in terms of the real orthogonal modes $\mathbf{u}^{(n)}$ of the body. The a_n are random coefficients determined by the field's source(s). In the presence of continuing sources, or dissipation, the a_n are slowly varying functions of time.

A statement that the field is diffuse is equivalent to stating that the a's are uncorrelated random numbers

$$\langle a_n\rangle = 0; \qquad \langle a_n a_m\rangle = 0; \qquad \langle a_n a_m^*\rangle = \delta_{nm}F(\omega_n) \tag{2}$$

where the brackets $\langle\rangle$ represent an (unspecified) average, and F represents the field's power density at frequency ω.

The auto-correlation of the field (1) is then simple to construct:

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$$\langle \mathbf{u}(\mathbf{X},t)\mathbf{u}(y,t+\tau)\rangle = \frac{1}{2}\mathrm{Re}\sum_{n=1}^{n=\infty} F(\omega_n)\mathbf{u}^{(n)}(\mathbf{x})\mathbf{u}^{(n)}(\mathbf{y}) \times \exp\{-\mathrm{i}\omega_n\tau\} \tag{3}$$

Except for the factor F (which filters and distorts the result) and the support that (3) has for negative τ, this is identical to the modal expansion for the Green's dyadic:

$$\mathbf{G}(\mathbf{x},\mathbf{y},t) = \mathrm{Im}\sum_{n=1}^{n=\infty} \mathbf{u}^{(n)}(\mathbf{x})\mathbf{u}^{(n)}(\mathbf{y})\exp\{-\mathrm{i}\omega_n\tau\}/\omega_n \tag{4}$$

A similar argument shows that diffuse waveforms (being temporal and spatial convolutions of the diffuse fields $\mathbf{u}(\mathbf{x},t)$ over detector area and time function) have correlations virtually identical to the impulse response function representing the signal that could travel between the detectors if one were used actively. It was concluded in [8] that the τ derivative of the cross-correlation of the diffuse signals in a pair of ultrasonic transducers should be identical to the pitch-catch waveform that is generated by pulsing one transducer and detecting with the other. A major caveat, it was noted, would be the effect of the diffuse field spectrum F. If F were proportional to $1/\omega^2$, the result is immediate; if F were some other smooth function of ω, the correlation function would be very similar to the pitch-catch signal. If F had complex structure, the correlation would be a highly distorted version of the pitch-catch waveform.

In order to confirm and illustrate these ideas, three ultrasonic pin-transducers (frequency range 0.1–0.9 MHz) were attached [8] with oil couplant to an irregular aluminum block ($\sim$2000 cm^3 volume) as sketched in Fig. 1. The direct pitch-catch waveform between two of the transducers was obtained using a conventional ultrasonic pulser and amplifier and digital data acquisition. This waveform was compared with another waveform obtained by cross-correlating the long-lived ($\sim$100 ms) noise-like diffuse signal that each of these transducers reported when the specimen was excited by the distant third transducer.

F (the spectrum of the diffuse field) was evaluated by measurements at the source of the diffuse field—the third transducer. Its effect on the correlation was removed by doing a kind of deconvolution. The long slow ring-down of the diffuse field, due to dissipation, was removed by certain compensating techniques that allowed us to make optimal use of the available information.

The two waveforms so obtained were similar, but not identical. The discrepancy was ascribed to the small amount of data available with which to construct the cross-correlation, being only 100 ms of diffuse field signal. Additional averaging over several distinct source positions improved the correspondence. Remaining discrepancies were ascribed to imperfect evaluation of F.

In a separate pair of publications [9,10], diffuse field correlations were re-visited, but using the diffuse field offered by thermal fluctuations. This has the advantage of eliminating uncertainties in F (classical thermal spectra are well known), and eliminating the need to compensate for dissipation (thermal fields have constant amplitude). The use of thermal fluctuations in the MHz range has one obvious disadvantage though—they are quite weak. This is readily established by considering that each mode of the structure has, in thermal equilibrium at room temperature, an expected energy of $kT \approx 4.2\times10^{-21}$ J. For typical solids, with mode counts below 1 MHz of about 300 modes/cm^3, this corresponds to energy densities of $\sim$10^{-12} J/m^3, rms strain ampli-

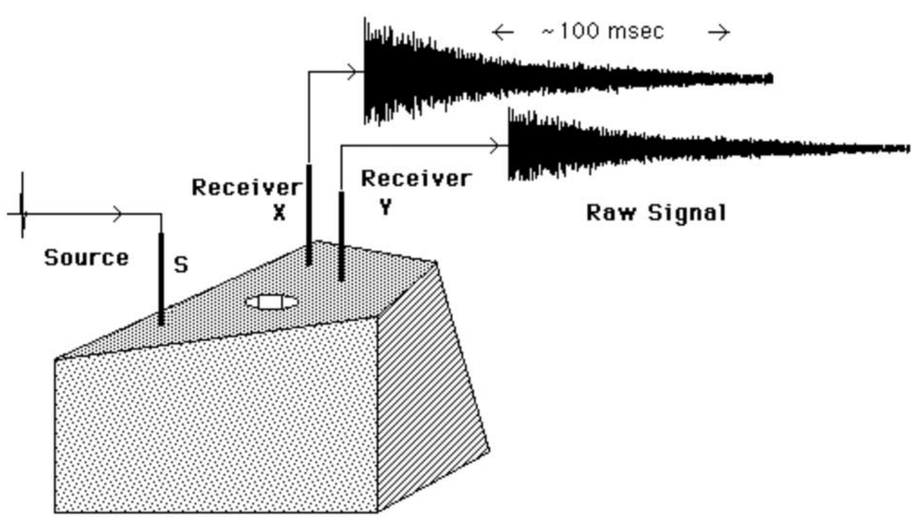

Fig. 1. A sketch of the three-transducer configuration. An impulsive source creates a long-lived diffuse field which is detected at receivers at x and y. After compensation for absorption, the cross-correlation between the detected signals is compared with another waveform, that which is detected at y after an impulsive excitation at x. Imperfect compensation for the non-flat spectrum produced by the source at s leads to an imperfect correspondence between these two waveforms.

tudes of $\sim 3 \times 10^{-12}$, and rms displacement amplitudes of $\sim 10^{-15}$ m. Nevertheless, as Fortunko established [11], modern piezoelectrics can be very sensitive and the best devices have noise floors close to the thermal limit.

Acoustic emission transducers are amongst the most sensitive, and two were chosen to illustrate these ideas. The Physical Acoustics "WD" transducer was found to have a noise level (after the noise is passed through a low-noise Panametrics battery-powered preamplifier) about 40% of which could be ascribed to thermal phonons in the sample. This figure was very different with other transducers, being only a few % in the case of a Valpey-Fisher pin-transducer and a Digital Wave B1025 transducer. For these tests (see Fig. 2) the auto-correlation of the thermal noise was compared with the waveform obtained in a pulse/echo configuration. In all cases the noise is dominated by the non-phononic component, presumably composed in part by electronic noise in the amplifier.

The electronic noise has, however, little correlation beyond a few microseconds (a figure which is probably related to the bandwidth of the amplifier.) The phononic noise has correlations out to time scales comparable to the absorption time in the specimen, being 100's of ms. Thus the auto-correlation is, at times $\tau >$ microseconds, entirely phononic. Correlations were constructed by capturing windows (typically a few ms in length) of noise, FFT'ing, squaring, and repeating, thus accumulating an estimate for the power spectral density of the noise $|FFT|^2$. In the case of the sensitive WD transducer, auto-correlations were found to converge reasonably well after only a few 10's of ms of actual noise data (requiring a few seconds of real time). The other transducers required substantially more data before convergence could be observed. Comparisons between the auto-correlation function so constructed and conventionally obtained pulse/echo signals showed

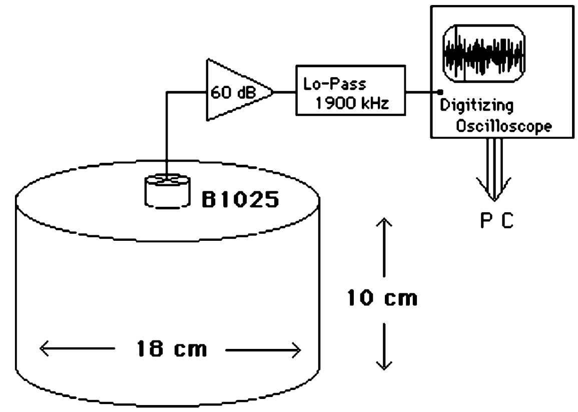

Fig. 2. The configuration employed for auto-correlation of thermal noise.

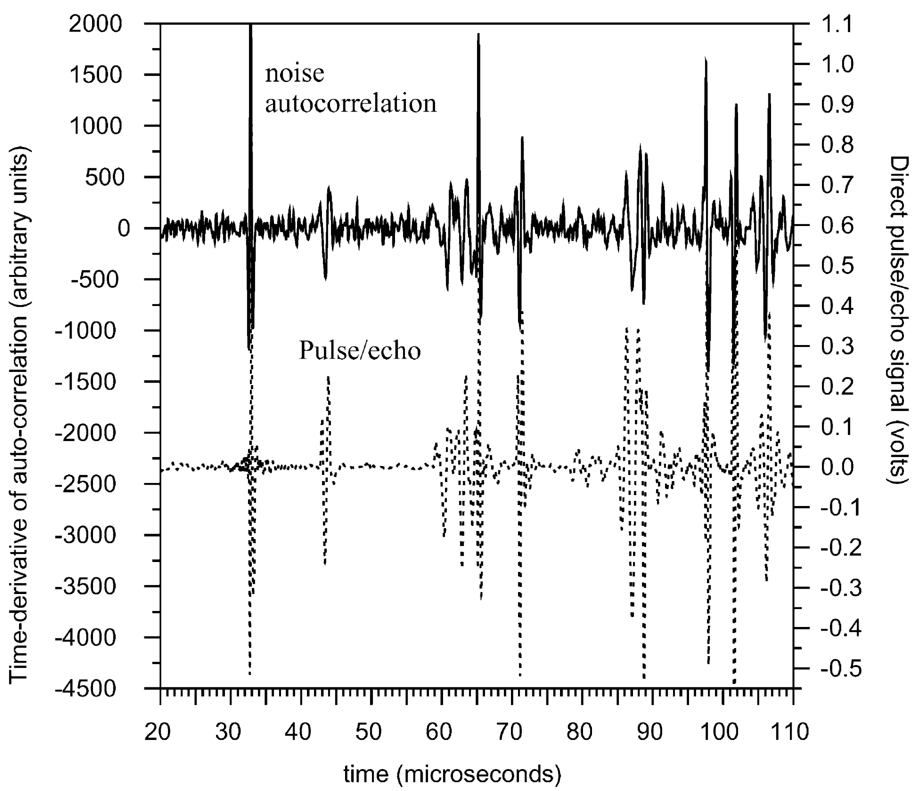

Fig. 3. The signal obtained by conventional pulse/echo with the Digital Wave B1025 transducer is compared with the time-derivative of the auto-correlation of its noise. Except for the greater noise level (due to finite averaging time when the auto-correlation is constructed) they are identical. They differ more at early times (not shown) where electronic noise has non-negligible correlations.

excellent agreement. One example is shown in Fig. 3. Except for residual noise due to finite averaging, and except for the expected differences at very short times (< microsecond) the waveforms were identical. Identity was preserved even at late times, greater than ms, for which ray paths are many meters.

The strength of the thermal correlations is, theoretically, proportional to temperature. By calibrating the transducer (using a broken capillary signal of known strength) and comparing with the amplitude of the noise correlations, it proved possible to confirm the thermal phonon origin of the noise. This allowed the set-up to be used as an ultrasonic bolometer in which the strength of the thermally excited ultrasonic waves is used to measure temperature. Temperatures were correctly inferred to be 293 ± 20 K.

That there is so much information in a diffuse field, and that the information is embodied in a simple identity between direct ultrasonic waveforms and correlations of diffuse fields, is remarkable. In spite of the simplicity of the mathematical proof and its basis in 19th century physics, this seems to have been overlooked to date. That thermally fluctuating elastic wave fields can do the job is also striking. Except for their weakness, they are in fact ideal because their spectra are so well known. That thermal fluctuations with displacement amplitudes of the order of femtometers can be detected is a testimony to the great sensitivity of modern piezoelectric devices.

There may be applications for these ideas. The chief virtue of the identity may be that it allows one to create waveforms without having a source. In the conventional ultrasonic regime, this may be of little utility; sources are not expensive. The economics may be different in other fields. Diffuse fields present in seismology (in the coda for example [6]) may allow construction of waveforms without the use of thumper trucks or explosives, and without depending on local seismic sources. Thermal fields at frequencies above a GHz may allow ultrasonic investigations of modern materials and microdevices for which conventional ultrasonics is not possible.

References

[1] R. Weaver, On diffuse waves in solid media, J. Acoust. Soc. Am. 71 (1982) 1608–1609;
R. Weaver, Laboratory studies of diffuse waves in plates, J. Acoust. Soc. Am. 79 (1986) 919–923.

[2] P.M. Morse, R. Bolt, Sound waves in rooms, Rev. Mod. Phys. 16 (1944) 69–150;
K.J. Ebeling, Statistical properties of random wave fields, in: W.P. Mason (Ed.), Physical Acoustics, vol. 17, Academic Press, New York, 1984, pp. 233–309.

[3] R. Weaver, Diffuse field decay for materials characterization, in: J.D. Achenbach, Y. Rajapakse (Eds.), Solid Mechanics Research for Quantitative Non-Destructive Evaluation, Martinus Nijhoff, The Hague, 1987, pp. 425–434;
R. Weaver, Indications of material character from the behavior of diffuse ultrasonic fields, in: J.F. Bussiere, J.-P. Monchalin, C.O. Ruud, R.E. Green (Eds.), Non-Destructive Characterization of Materials II, Plenum Press, New York, 1987, pp. 689–696;
M. Paul, B. Haberer, W. Arnold, Materials characterization at high temperatures using laser ultrasound, Mater. Sci. Eng. A 168 (1993) 87–92;
A. Moreau, M. Lord, D. Levesque, M. Dubois, J.F. Bussiere, Laser Ultrasonics absorption measurement in low carbon steels, J. Alloys Compd. 310 (2000) 427–431;
H. Willems, A new method for measurement of ultrasonic absorption in polycrystalline materials, in: D.O. Thompson, D.E. Chementi (Eds.), Review of Progress in Quantitative NonDestructive Evaluation, vol. 6A, Plenum Press, New York, 1987, pp. 473–481.

[4] S.M. El-Sherbiny, Determination of the attenuation coefficient of a scattering medium from the ultrasonic back scatter, Acustica 48 (1981) 10–14;
F.J. Margetan, T.A. Gray, R.B. Thompson, A technique for quantitatively measuring microstructurally induced ultrasonic noise, in: D.O. Thompson, D.E. Chimenti (Eds.), Review of Progress in Quantitative NDE, vol. 10, Plenum Press, New York, 1991, pp. 1721–1728;
M.D. Russell, S.P. Neal, Grain noise power spectrum estimation for weak scattering polycrystalline materials using experimentally estimated backscatter coefficients: I normal incidence, and II oblique incidence, Ultrasonics 32 (1994) 163–171, 173–180.

[5] C.B. Guo, P. Holler, K. Goebbels, Scattering of ultrasonic waves in anisotropic polycrystalline metals, Acustica 59 (1985) 112–120;
R. Weaver, Ultrasonic diffuse field measurements of grain size, in: H.L.M. dos Reis (Ed.), Non-Destructive Testing and Evaluation in Manufacturing and Construction, Hemisphere, New York, 1990;
R. Weaver, K. Green,W. Sachse, Diffuse Ultrasound in Polycrystals: Measurements in Brass and Steel, Proceedings of the Conference "Ultrasonics International 1991," Le Toquet, France, July 1991;
R.L. Weaver, W. Sachse, Diffusion of ultrasound in a glass bead slurry, J. Acoust. Soc. Am. 97 (1995) 2094–2102;
G. Lamouche, M. Lord, A. Moreau, unpublished presentation at the Dec. 2000 meeting of the Acoustical Socioety of America.

[6] R. Hennino, N. Trégourès, N.M. Shapiro, L. Margerin, M. Campillo, B.A. van Tiggelen, R.L. Weaver, Observation of equipartition of seismic waves, Phys. Rev. Lett. 86 (2001) 3447–3450;
N. Tregoures et al., Multiple scattering of seismic waves, Ultrasonics, this issue.

[7] R. Weaver, Spectral statistics in elastodynamics, J. Acoust. Soc. Am. 85 (1989) 1005–1013;
R.L. Weaver, Wave chaos in elastodynamics, in: P. Sebbah (Ed.), Waves and Imaging through Complex Media, Proceedings of the International Physics School on Waves and Imaging through Complex Media, Cargese France, Kluwer Academic Publishers, Dordrecht, 2001, pp. 141–186;
C. Ellegaard, T. Guhr, K. Lindemann, J. Nygård, M. Oxborrow, Symmetry breaking and spectral statistics of acoustic resonances in quartz blocks, Phys. Rev. Lett. 77 (1996) 4918–4921;
C. Ellegaard, T. Guhr, K. Lindemann, H.Q. Lorensen, J. Nygård, M. Oxborrow, Spectral statistics of acoustic resonances in aluminum blocks, Phys. Rev. Lett. 75 (1995) 1546–1549.

[8] R.L. Weaver, O.I. Lobkis, On the emergence of the Green's function in the correlations of a diffuse field, J. Acoust. Soc. Am. 110 (2001) 3011–3017.

[9] R.L. Weaver, O.I. Lobkis, Ultrasonics without a source, thermal fluctuation correlations at MHz frequencies, Phys. Rev. Lett. 87 (2001) 134301.

[10] R.L. Weaver, O.I. Lobkis, Elastic wave thermal fluctuations, ultrasonic waveforms by correlation of thermal phonons, J. Acoust. Soc Am., submitted for publication.
[11] C.M. Fortunko, E.S. Boltz, Comparison of absolute sensitivity limits of various ultrasonic and vibration transducers, in: Proceedings of the Seventh International Conference on Nondestructive Characterization of Materials, Prague, June 19–24, Plenum Press, New York, 1996;
E.S. Boltz, C.M. Fortunko, Absolute sensitivity of various ultrasonic transducers, in: B.R. McAvoy (Ed.), Proceedings of the 1995 Ultrasonics Symposium, IEEE, New York, 1995.