Verification of Rayleigh-wave dispersion characteristics using synthetic microtremors

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SUMMARY

An attempt to verify the characteristics of experimental Rayleigh-wave dispersion curve from passive microtremor measurements is carried out through synthetic microtremors. As case study, array microtremor measurements were performed at an Alpine site. The average experimental vertical Rayleigh dispersion curves obtained by the *f-k* spectral method was used to estimate subsoil models by random search (Monte Carlo inversion). Also, the predominant frequency of experimental H/V spectral ratio was calculated to improve the inversion process. Several researchers have reported that the fundamental frequency of theoretical H/V spectral ratio or the predominant frequency of experimental H/V spectral ratio would represent an approximate threshold for the lowest frequency of experimental vertical Rayleigh dispersion curve. However, the highest velocity of the experimental Rayleigh dispersion curve occurred at a frequency higher than the value of the experimental H/V predominant frequency. In consequence, this experimental high velocity would not correspond to the fundamental Rayleigh dispersion curve. For verifying purposes, synthetic microtremors using a similar array to the one used at the site, were calculated for one of the subsoil models estimated through Monte Carlo inversion. The synthetic vertical Rayleigh dispersion curve was in good agreement with the experimental one, verifying the above assumptions.

Introduction

According to empirical results reported by several researchers (e.g., Bonnefoy-Claudet *et al*., 2006), the experimental spectral H/V predominant frequency or the theoretical H/V fundamental frequency would represent an approximate threshold for the experimental fundamental Rayleigh modal curve. Hence, if we assume this correlation as true, and if the highest velocity of the experimental Rayleigh dispersion curve occurs at a frequency higher than the value of the H/V predominant frequency, then, this experimental high velocity would not correspond to the fundamental Rayleigh modal curve and it would therefore be related to higher mode superposition.
To verify these assumptions, microtremor measurements recorded at an Alpine site are analyzed as a case study, and compared with synthetic microtremors both in terms of Rayleigh dispersion curves and the modified H/V spectral ratios (Tokimatsu, 1995). Also, a correlation between the value of the predominant frequency of the classical H/V spectral ratio (Nakamura, 1989) and the highest velocity of the synthetic horizontal dispersion curve is discussed.

Estimation of observed surface wave dispersion characteristics

Synchronized microtremor measurements with a sampling frequency of 100 Hz were performed (Fig. 1). Twelve sets of 40 s vertical component records were considered simultaneously. The observed Rayleigh dispersion characteristics were estimated using the *f-k* spectral method (beam forming) through a software developed by the Natural Research Institute for Earth Science and Disaster Prevention of Japan. Figure 2 shows the average observed Rayleigh dispersion curve of the 12 sets (red dots) that presents a frequency band of 8.4 – 20 Hz. Also, the standard deviation for each frequency was calculated and used in the inversion.
Moreover, the predominant frequencies of the modified-H/V spectral ratio and classical one were calculated to find some correlations with the lowest frequency observed in surface waves dispersion curves. There was no one clear predominant peak within the first predominant frequency range (5.3–6.3 Hz) of the modified-H/V spectral ratio (Fig. 3-a), and the correlation between the value of this predominant frequency and the lowest frequency of observed Rayleigh dispersion of 8.4 Hz was not obtained in this site. This fact may indicate that high velocities at low frequencies of the experimental Rayleigh dispersion curve do not correspond to the fundamental mode. Concerning the classical-H/V spectral ratio, a predominant peak around 5.5 Hz was obtained (Fig. 3-b).
For the inversion process, a 4-layer model overlying a half-space was used (Table 1). A Monte Carlo inversion was performed using a random search on three parameters (thickness, S-wave velocity and Poisson's ratio) in all layers. The other properties of layers, such as the P-wave velocity and the density, were calculated after defining the values of the 3 random parameters (Tokeshi *et al*, 2008). Figure 4 shows the 3 subsoil models estimated after one million trials. Figure 2 shows the theoretical fundamental Rayleigh-modes of the obtained models, which only satisfy 10 points of the experimental Rayleigh-dispersion curve and the theoretical 1st higher Rayleigh-mode, suggesting that the experimental Rayleigh dispersion curve is influenced by higher Rayleigh-modes at low frequency. To demonstrate this fact, synthetic microtremors were simulated.

Mathematical model used for calculating synthetic microtremors

The synthetic microtremors were calculated according to the technique developed by Tokeshi *et al*. (2000) for one of the 3 subsoil models estimated through Monte Carlo inversion (Table 2). An infinite surface divided into small squares of 200 m side length was used (Fig. 5).
Six stations forming a similar array to the one used at the site (Fig. 1) were set in the model of Figure 5, with the central station (S01) placed at the centre of the model. A total of 8192 three

dimensional sources of pseudo-Dirac type (delta function) with predetermined constant amplitude was randomly distributed at the centre of the elements at a random time between 0 and 81.91s. Only sources whose distances to stations were within 1000 m radius were considered in the calculation of corresponding Green's functions. The 3 components of the synthetic microtremors at each station were calculated with 100 Hz sampling frequency. The simulation was parametrically performed by using the ratio of forces, R_F, between the horizontal force (Qh = variable) and the vertical force (Qv = 1) of the input source, which was introduced by Tokeshi *et al.* (2000). For discussing the characteristics of the observed surface wave dispersion curves, synthetic microtremors for $R_F = 1$ were performed.

Characteristics of surface wave dispersion curves from synthetic microtremors

The synthetic surface wave dispersion curves were calculated by the *f-k* spectral method. Two sets of 40 s each frame were analyzed. In case of $R_F = 1$, only Rayleigh-waves should be predominant in the vertical component synthetic dispersion curves, and Love-waves should be predominant in the horizontal one. Figure 6-a displays the comparison between observed and synthetic vertical dispersion curves. The synthetic vertical dispersion curve is in good agreement with the experimental one, verifying thus, the soil profile estimated from inversion. The highest apparent velocity at the lowest frequency of synthetic- and experimental-Rayleigh dispersion curve is overestimated by the *f-k* spectral method. Excepting this frequency, both dispersion curves approach to the theoretical 1st higher Rayleigh-mode up to 11.5 Hz, and then shift to the theoretical fundamental Rayleigh-mode at higher frequencies. Figure 6-b displays the synthetic horizontal dispersion curves that approach to the theoretical fundamental Love–mode, meaning that the Love-waves in the horizontal components of synthetic microtremors are predominant.
The modified H/V spectral ratio for $R_F = 1$ in Fig. 7-a shows two predominant frequencies (5.8 and 7.2 Hz). The larger frequency of 7.2 Hz is displayed as vertical dash lines in Fig. 6-a, which is also lower than the frequency for the highest synthetic- and experimental Rayleigh phase velocity obtained by the *f-k* spectral method. Figure 7-b shows the classical H/V spectral ratio, whose predominant frequency is around 5.3 Hz. This value is displayed as vertical dash line in Fig. 6-b, which seems to be the threshold for the synthetic Love dispersion curve. Similar general results were obtained for the other 2 soil profiles estimated from inversion and also, the experimental Love dispersion curves were calculated and compared with the synthetic one (not shown here).

Conclusions

The Rayleigh-wave dispersion curve of one subsoil model estimated by Monte Carlo inversion was successfully verified through the simulation of synthetic microtremors. The synthetic vertical Rayleigh dispersion curves were in good agreement with the experimental one obtained at the site. Both dispersion curves, which were obtained by *f-k* spectral method, approached to the theoretical 1st higher Rayleigh-mode at frequencies lower than 11.5 Hz, and then shifted to the theoretical fundamental Rayleigh-mode at frequencies higher than 11.5 Hz.
The value of the predominant frequency of modified H/V spectral ratio was lower than the lowest frequencies in both Rayleigh dispersion curves confirming that this parameter can be used to recognise the influence of higher modes in experimental dispersion curves. Also, the highest velocity of the synthetic Love dispersion curve occurred at a frequency quite close to the predominant frequency of classical H/V spectral ratio, which could be used as possible threshold for this type of surface wave. Further tests are needed to verify this possible relationship.

Table 1 *Parameter ranges used for random search in Monte Carlo inversion.*

Layer	ν_o	ν_f	Vs_o (m/s)	Vs_f (m/s)	H_o (m)	H_f (m)
1	0.25	0.49	100	600	1	50
2				1000		100
3				1500		100
4				2200		100
Half-space	V_P = 3800 m/s		V_S = 2500 m/s		Infinite	

Table 2 *Physical parameters of subsoil model used for synthetic microtremors simulation.*

Layer	Thickness (m)	Density (t/m3)	V_P (m/s)	V_S (m/s)	Q_P & Q_S
1	9	2.04	1875	401	20
2	20	1.98	1665	817	40
3	11	2.36	3355	1322	65
4	41	2.36	3370	1836	90
Half-space	Infinite	2.5	3800	2500	125

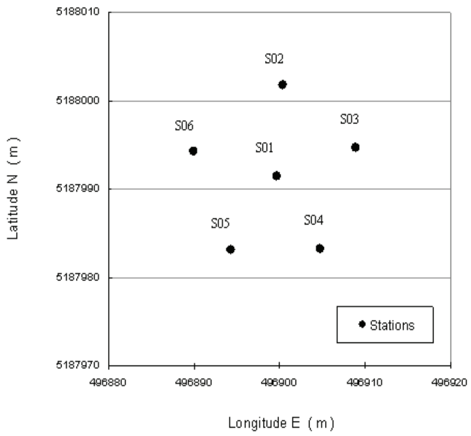

Figure 1 *Array of microtremor measurements*

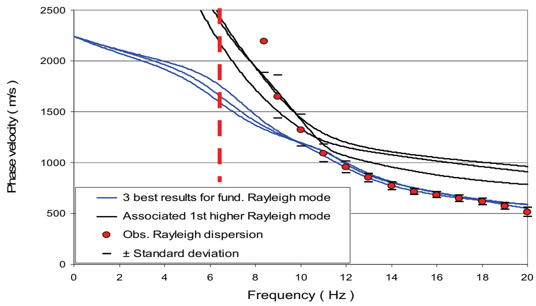

Figure 2 *Comparison of observed Rayleigh dispersion curve and theoretical Rayleigh-modes.*

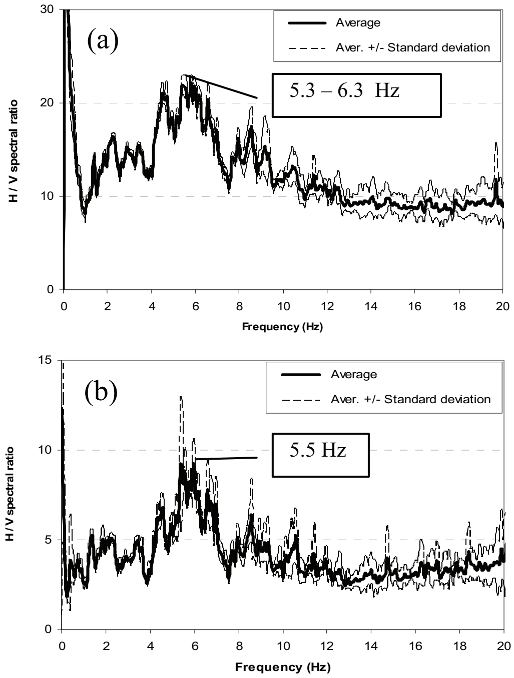

Figure 3 *(a) Modified and (b) classical H/V spectral ratios obtained from microtremor records.*

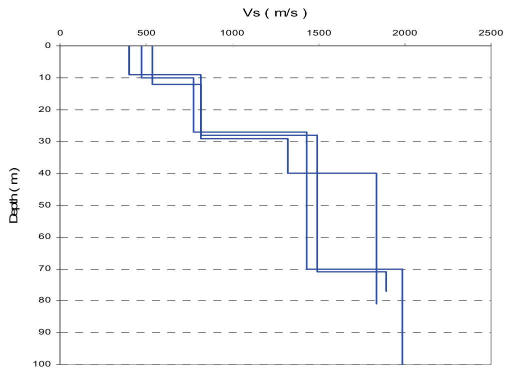

Figure 4 *Vs final models estimated from fundamental Rayleigh-mode*

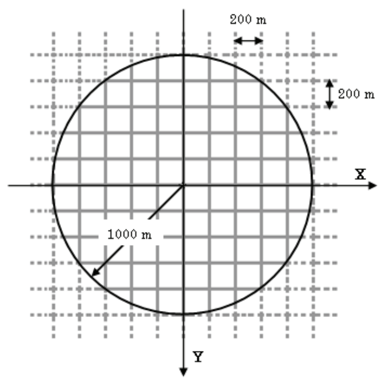

Figure 5 *Mathematical model used for calculating synthetic microtremors.*

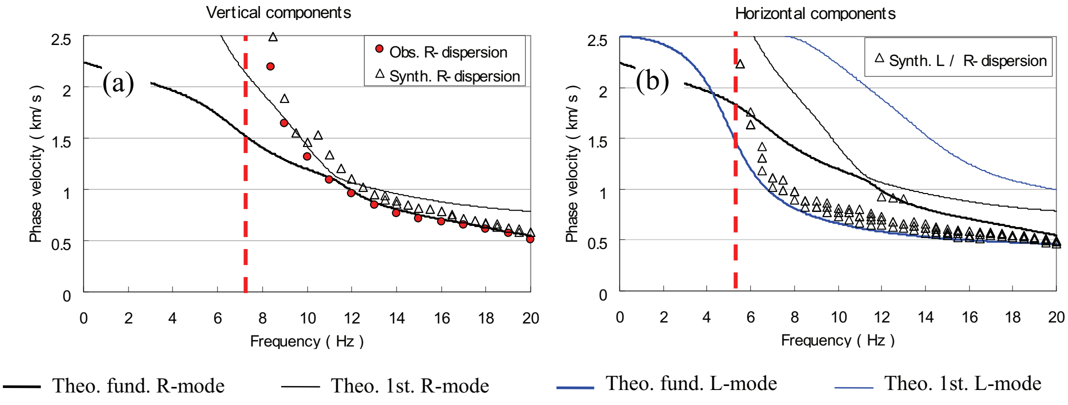

***Figure 6** Observed and synthetic Rayleigh- / Love-dispersion curves (R_F =1) obtained by f-k spectral analysis and comparison with the theoretical one.*

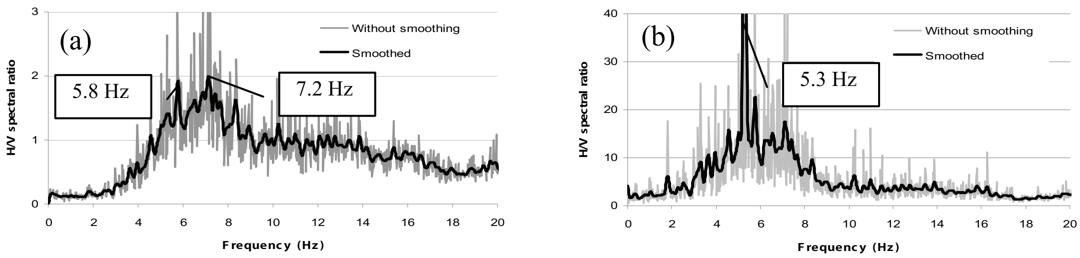

***Figure 7** (a) Modified and (b) classical H/V spectral ratios obtained from synthetic microtremors for R_F =1.*

Acknowledgements

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