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Inter Event Times of Fluid Induced Seismicity

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SUMMARY

We show that the temporal occurrence of seismic events induced by borehole fluid injections is a Poissonian process. We demonstrate this by analyzing the distribution of inter event times between successive events induced in six different case studies. We show that during stationary phases of seismicity the occurrence of events follows a homogeneous Poisson process, while the temporal organization of a complete seismic sequence can be described according to a non homogeneous Poisson process. These results prove the independency of fluid induced events. The underlying process leading to seismic events is hence not influenced by the occurrence of an event. Our results build a basis for upcoming seismic risk studies in geothermal and hydrocarbon industry.

Introduction

In the last years, the analysis of waiting times between subsequent earthquakes has been the subject of various works [see e.g. Bak et al. (2002), Shcherbakov et al. (2005), Molchan (2005), Corral (2006)]. Most of these works aim to identify the statistical processes describing the temporal distribution of events. Particularly with respect to the design of seismic risk studies the understanding of these processes is important to establish fundamental probabilistic models. Although the seismic risk resulting from fluid injections into geothermal and hydrocarbon reservoirs is an up to date topic the waiting times between seismic events induced by these injections have not yet been studied. Because induced events seem to have a complex distribution the question arises, if a basic statistical process can be found, which describes the distribution of fluid induced seismicity in time? To answer this question we analyze the distribution of waiting times in seismic sequences induced by fluid injections into geothermal reservoirs in six different case studies.

We start with a short description of the basic statistic of a Poisson process, that is, a process consisting of independent events. We then analyze inter event times during phases of stationary fluid induced seismicity and compare it to inter event time distribution resulting from a homogeneous Poisson process. In the next section the complete induced seismic sequences are analyzed and compared to a non homogeneous Poisson process. We finalize our work with a short concluding discussion of the results.

Theory

A sequence of temporally uncorrelated events, that is, a sequence including only events that do not affect the occurrence probability of later events, is described according to a Poisson process (e.g. radioactive decay). In the most simple case of a Poisson process with constant event rate λ the process is called Homogeneous Poisson Process (HPP). The occurrence probability $P(n, \lambda, t)$ to have n events in the time interval $(0, t)$ is then equal to:

$$P(n, \lambda, t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}. \quad (1)$$

The most appropriate way to demonstrate that events in a sequence obeys the Poisson distribution is to consider the time between subsequent events, which are named inter event times, waiting times or inter occurrence times in different works. The probability that the time between two events is smaller than Δt is $1 - P(0, \lambda, \Delta t)$, where $P(0, \lambda, \Delta t)$ is the probability that no event occurs in the time interval $0 < t < \Delta t$. The derivative $\frac{\partial}{\partial \Delta t}(1 - P(0, \lambda, \Delta t))$ gives us the probability density function (pdf) of inter event times Δt:

$$pdf(\Delta t) = \lambda e^{-\lambda \Delta t}. \quad (2)$$

To allow a comparison between data sets with different event rates λ, it is reasonable to analyse the distribution of normalized inter event times $\Delta\tau$ given by $\Delta\tau = \Delta t \lambda$ [see Corral (2006)]. In this way $pdf(\Delta\tau)$ is proportional to $e^{-\Delta\tau}$. We now analyse the distribution of normalized inter event times $\Delta\tau$ between subsequent seismic events induced during borehole fluid injections.

Stationary seismicity: A Homogeneous Poisson Process (HPP)

We here analyze six catalogs of fluid induced seismicity collected during different injection experiments. Often event catalogs are incomplete due to undetected small magnitude events. Therefore, we select only events with magnitude $m > m_c$, where the magnitude m_c is the cut of magnitude below which the Gutenberg Richter relation [Gutenberg and Richter (1954)] collapses. Figure 1 shows the event rates of the six resulting catalogs. For the first analysis of inter event time distribution we select phases of stationary seismicity, that is, phases of approximately constant event rate (linearly increasing cumulative event number). The selected stationary phases are marked in Figure 1. Each phase consists of at least 550 events.

We now calculate normalized inter event times $\Delta\tau_i = (t_i - t_{i-1})\,\hat{R}$ between successive events, where $\hat{R}$ is the mean seismicity rate during a stationary phase. Because we are dealing with inter event times that differ in various dimensions, we analyze the distribution of inter event times in logarithmical binned

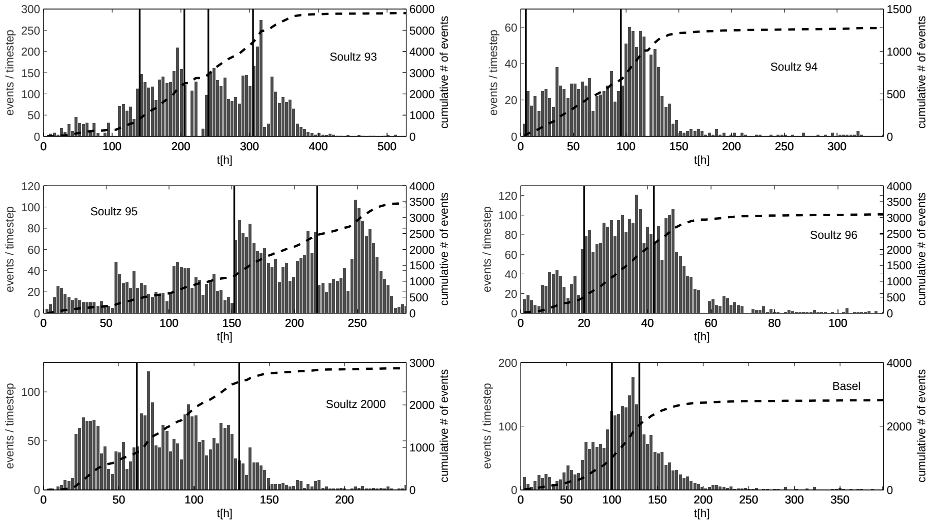

Figure 1 *Seismic activity induced by six different borehole fluid injections. The Figures show the number of events per time step (grey bars) and the cumulative number of events (dashed black line). Furthermore, stationary phases of seismicity are marked. The stationary phases which we have selected for the inter event time analysis are located between the black vertical lines.*

time intervals. More precisely, we analyze intervals given by $[\tau, \tau + d\tau] = [1, c], [c, c^2], [c^2, c^3]$... [see also Corral (2006)]. In a logarithmic time scale these intervals are equally spaced. We then count the number of inter event times falling in a time interval. Afterwards we divide the number of counts by the length of the interval. In this way the probability density function $pdf(\Delta\tau)$ is determined. In the last step we normalize the pdf according to $\int_0^\infty pdf(x)dx = 1$.

To measure the extend of natural statistical fluctuations in the distribution, we create 550 events distributed according to a HPP with length $T = 100h$. This corresponds to the statistic of the stationary phase with the smallest number of events. Afterwards, we calculate the probability density function of inter event times for synthetic events in the same way as for real ones.

In Figure 2 the result of the inter event time analysis is presented. The Figure shows the $pdf(\Delta\tau)$ for the seven stationary phases, the pdf calculated for the synthetic events and the theoretical pdf $e^{-\Delta\tau}$ of the HPP. Apart from negligible small fluctuations all plotted curves coincide over the whole value range of $\Delta\tau$. The unique exponential trend proves that fluid induced events during stationary phases are distributed according to a HPP with rate parameter $\hat{R}$. Thus, regardless of a considered time window fluid induced events during stationary seismic phases are independent.

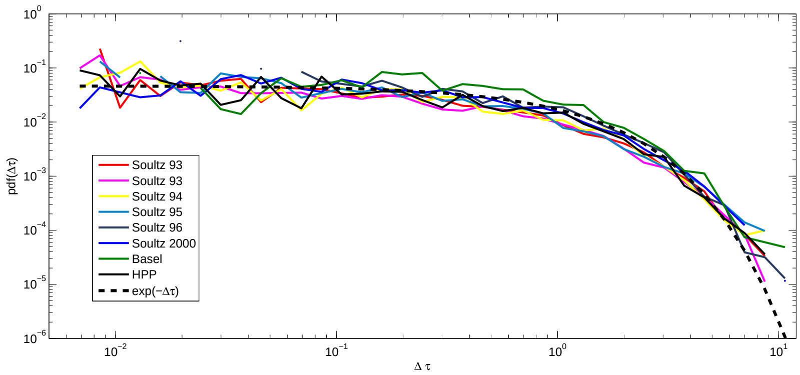

Figure 2 *Inter event time analysis for stationary seismic phases (highlighted in Figure 1). The plot shows the pdf of rescaled inter event times $\Delta\tau = \Delta t\hat{R}$ for the stationary phases (colored lines) together with the $pdf(\Delta\tau)$ calculated for 550 Poisson distributed events (solid black line) and the theoretical pdf $e^{-\Delta\tau}$ of the HPP (dashed black line).*

Complete induced seismic sequences: A Non Homogeneous Poisson Process (NHPP)

Our analysis of events during stationary phases of induced seismicity has shown that these events are distributed according to a HPP. What reveals an analysis of a complete seismic sequence with time dependent event rate $R(t)$? If events during a complete seismic sequence are also distributed according to a Poisson process their occurrence has to follow a NHPP with time dependent event rate $\lambda(t) = R(t)$. In this case the probability to induce n events by a fluid injection in the time interval $(0, t)$ would be given by:

$$P(n, R(t), t) = \frac{\left[\int_0^t R(t')dt'\right]^n}{n!} exp\left[-\int_0^t R(t')dt'\right]. \quad (3)$$

We now compare the inter event time distribution of events included in a whole seismic sequence to syntetic inter event time calculated for events simulated according to a NHPP with time dependent event rate $\lambda(t) = R(t)$. Therefore, we first create events following a homogeneous Poisson process with event rate $\lambda \geq R(t)$, for $0 \leq t \leq T$, where T is the total time of a sequence. After creating events according to a HPP we use the approach known as the thinning or rejection method [see Ross (2002), Bratley et al. (1987)] to select all arrivals corresponding to a NHPP with rate parameter $R(t)$. This method uses the following procedure. The ith event of the simulated HPP is accepted with probability $R(t_i)/\lambda$. Here, t_i is the occurence time of the ith event. For all events in the HPP this is done independently of all other events. We then obtain a sequence $a_1, a_2, \ldots, a_n$ of accepted events. This sequence now contains events according to a NHPP with event rate $R(t)$.
Figure 3 shows a result of the described simulation for the Soultz 93 case study. The black bars in the show the temporal distribution of events created according to a HPP with the maximum event rate identified in the Soultz 93 data set (see Fig. 1). The red bars show the distributuion of remaining events after thinning corresponding to the Soultz 93 event rate.

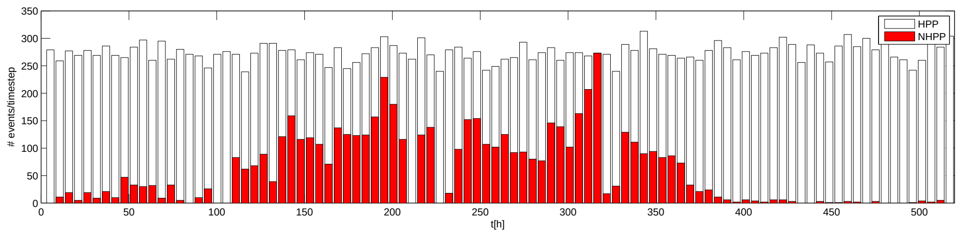

***Figure 3** Simulated event rate of the Soultz 93 injection. The event rate is simulated according to a NHPP. Black bars: Event rate of the initially simulated HPP. Red bars: NHPP resulting from thinning according to the Soultz 93 event rate shown in Figure 1.*

We now perform this simulation for all six case studies shown in Figure 1 and calculate the pdf of inter event times. Figure 4 shows the comparison of simulated and real inter event time distributions. The probability density functions coincide over the whole value range of Δt identified in the seismic sequences. Thus, the events are distributed according to a NHPP. From this it follows that fluid induced events occure independently from each other. Additionally, the theoretical distribution resulting from event simulation according to a HPP is shown for comparison in Figure 4.

Conclusions

We have shown that events induced by borehole fluid injections are distributed according to a Poisson process. During stationary periods of seismicity the occurrence of events follows a homogeneous Poisson process. A complete seismic sequence induced by a fluid injection contains events distributed corresponding to a non homogeneous process. This means, that an induced event does not influence the underlying process leading to the occurrence of later events. Or in other words, the occurrence of an event does not modify the occurrence probability of later events. The probability to induce a certain number of events by injection of fluids can hence be calculated according to Eq. 3. This result opens up a possibility to create synthetic event sets with internal temporal organization corresponding to real events. By calculating the seismicity rate $R(t)$, which can be done with the results obtained by Parotidis

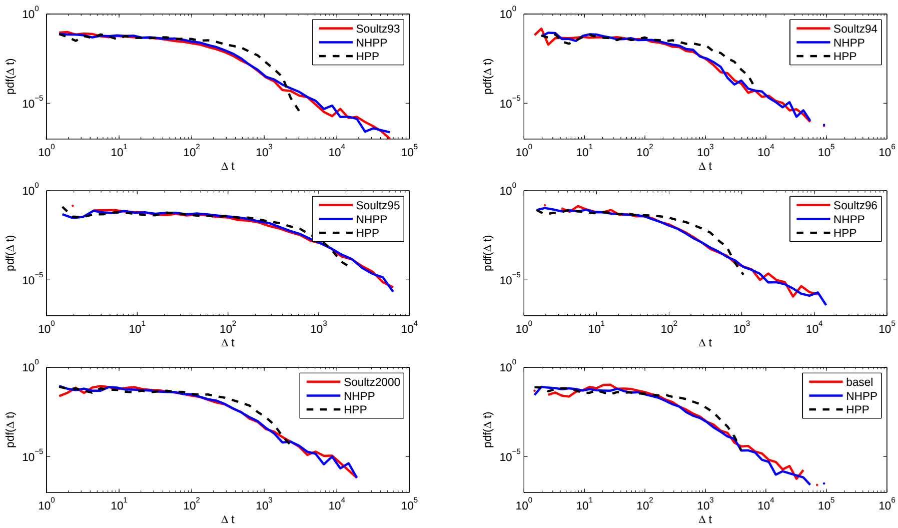

Figure 4 *Distribution of inter event times in complete seismic cycles. The Figure shows inter event time distributions of real events (red solid line), events simulated according to a NHPP (blue solid line) and events simulated according to a HPP (black dashed line).*

and Shapiro (2004) and Langenbruch and Shapiro (2009), seismic risk studies can be organized. By combining Eq. 3 with the Gutenberg Richter relation also the risk of inducing events above a certain magnitude can be calculated.
Our obtained results verify the model used in various works of Shapiro et al. [see e.g. Shapiro et al. (2007), Rothert and Shapiro (2007)]. In these works the assumed underlying process leading to the occurrence of seismic events is pore pressure diffusion. Furthermore, they assume a medium containing pre-existing fractures with randomly distributed fracture strength. This fracture strength is represented by a critical pore pressure, which has to be exceeded to trigger an event.

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